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Quantification

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Abstract

Medieval authors approached the semantic phenomenon now known as quantification essentially by means of the concept of supposition, more specifically the different modes of personal supposition. The modes of personal supposition were meant to codify the quantificational behavior of what we now refer to as quantifier expressions, and what the medievals referred to as syncategorematic terms. Perhaps the best way to understand the medieval approach to these quantifier expressions by means of the notion of supposition is as a two-step procedure that explicates their meaning and semantic behavior. First, the syntactical structure of the proposition, that is, the presence and order of its syncategorematic terms, determines the kind of personal supposition that each categorematic term has. Then, the semantic definitions of each mode of personal supposition determine the effect of quantifying syncategoremata over the quantity of objects involved in the assertion of a proposition. This entry discusses both groups of rules, and the contrasting thirteenth and fourteenth century approaches. The former is based on the verification of propositions and focuses on the semantics of quantifier expressions taken individually; the latter focuses on the inferential relations of ascent and descent between propositions with quantifying syncategorematic terms and singular propositions of the form "This a is b," and on the study of the global quantificational effect of syncategorematic terms in wider propositional contexts.

The phrase "medieval theories of quantification" is, properly speaking, an anachronism; medieval authors never used the term "quantification" in this sense, and even though they did treat semantic phenomena similar to what we now refer to as quantification, their theories differ from modern theories of quantification in significant aspects – to the point that this approximation may even be unwarranted (Matthews 1973). Nevertheless, their treatments of such phenomena are often insightful and sophisticated, justifying thus that we consider them from the viewpoint of modern theories of quantification, but provided that the term "quantification" be understood very broadly.

Broadly understood, quantification can be defined as a construct or procedure by means of which one specifies the quantity of individuals of the domain of discourse that apply to or verify a given statement. Typical quantifier expressions are "Some," "All," "None," and they usually determine the quantity of individuals involved in an assertion. Medieval authors approached quantification and quantifier expressions essentially by means of the concept of supposition, more specifically the different modes of personal supposition.

Besides supposition, they also treated quantificational phenomena from the vantage point of their theories of syllogisms, following the traditional Aristotelian approach. However, it is widely acknowledged that medieval authors did not contribute much to the development of Aristotle's theory of syllogisms for assertoric propositions, and that their main contributions concern modal syllogisms. Therefore, the innovations proposed by medieval authors with respect to quantification are not to be found in their theories of syllogisms, but rather in this typical medieval development, theories of supposition.

The different modes of personal supposition are indeed the closest medieval counterpart of our theories of quantification. The modes of personal supposition were meant to codify the quantificational behavior of what we now refer to as quantifier expressions, and what the medievals referred to as syncategorematic terms. Such analyses can be found in virtually every later medieval textbook in logic, but for reasons of space I shall focus on three representative texts: William of Sherwood's *Introduction to Logic*, William of Ockham's *Sum of Logic* (part I), and Buridan's *Treatise on Supposition*.

Perhaps the best way to understand the medieval approach to these quantifier expressions by means of the

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notion of supposition is as a two-step procedure that explicates their meaning and semantic behavior. First, the syntactic structure of the proposition, that is, the presence and order of its syncategorematic terms, determines the kind of personal supposition that each categorematic term has. Then, the semantic definitions of each mode of personal supposition determine the effect of quantifying syncategoremata over the quantity of objects involved in the assertion of a proposition.

In other words, the various theories of supposition presented by medieval authors typically have two groups of rules for the modes of personal supposition: the syntactic rules mapping terms in the propositional contexts created by quantifier expressions into modes of personal supposition; and the semantic rules mapping modes of personal supposition into specific semantic behaviors (see Ashworth 1978). To illustrate this, let us first discuss the four Aristotelian classes of categorical propositions: universal affirmative (A), particular affirmative (I), universal negative (E), and particular negative (O); and provide the two kinds of rules for these propositional forms. (Notice that, even at early stages of its development, supposition theory already recognized a wide variety of quantifier expressions – unlike modern quantification theory, which started out with the existential and universal quantifiers and only later developed into a theory of generalized quantifiers. Notice also that, for medieval logicians, following Aristotle, all affirmative propositions have existential import, existential and universal propositions alike.)

(A) Every *a* is *b*.
(E) No *a* is *b*.
(I) Some *a* is *b*.
(O) Some *a* is not *b*.

Syntactical Rules

The syntactical rules for these four propositional forms are easily enumerable, but in practice the enumeration of rules becomes very long when authors attempt to cover a wider range of propositional forms. The rules below can be found in all of our authors (Sherwood, *Introduction to Logic*, §5.13.1; Ockham, *Summa logicae I* chaps. 71–74; Buridan, *Summulae de suppositionibus*, chaps. 4.3.7 and 4.3.8.):

- The positive universal syncategorema "Every" (*omnis*) causes the term immediately following it to have confused and distributive supposition (*a* in (A)), and the term mediately following it to have merely confused supposition (*b* in (A)).
- A negative term, "No" (*nullus*) or "not" (*non*), causes all terms to its right to have confused and distributive supposition (*a* and *b* in (E) and *b* in (O)).
- The particular universal syncategorema "Some" (*alliquid*) causes the term immediately following it to have determinate supposition (*a* in (I) and (O)).
- In the absence of syncategorematic terms immediately preceding a term, and of universal terms affecting a term mediately, a term has determinate supposition (*b* in (I)).

Semantic Rules

Authors account for the semantic behavior of the various modes of personal supposition in different ways, in particular with a clear cleavage between thirteenth and fourteenth century approaches. In the thirteenth century, with Peter of Spain, William of Sherwood, and Lambert of Auxerre, there was a tendency toward defining the modes of personal supposition in terms of the verification of the proposition or the supposition of its terms:

- Supposition is determinate when the locution can be expounded by means of some single thing, which is the case when the word supposits for some single thing. (Sherwood, *Introduction to Logic*, §5.2.)
- Supposition is distributive when [the word] supposits for many in such a way as to supposit for any. (Sherwood, *Introduction to Logic*, §5.2.)
- A term has merely confused supposition in a categorical proposition when it can be taken there for several of its supposita, not necessarily for all. (For want of a satisfactory formulation of merely confused personal supposition in our authors, this is Parsons' (1997:45) "generic" version.)

By contrast, in the fourteenth century with Walter Burley, William of Ockham, and John Buridan, it became customary to define the modes of personal supposition in terms of "ascent and descent," that is, in terms of the inferential relations that do or do not obtain between a proposition and the singular propositions falling under it, of the form "This *a* is *b*" (see Priest and Read 1977; Spade 1996:chap. 9).

Let (S) and (Q) stand for any syncategorematic terms, and the general form of a proposition P be "(Q) *a* is (S) *b*." The generic definitions of the modes of personal supposition in terms of ascent and descent can be formulated as (see Ockham *Summa logicae* I, chap. 70; Buridan, *Summulae de suppositionibus*, chaps. 4.3.5 and 4.3.6.):

- A term *a* has determinate supposition in P ⇒ A disjunction of propositions of the form "This

a is (S) *b*" can be inferred from P but a conjunction of propositions of the form "This *a* is (S) *b*" cannot be inferred from P.

- A term *a* has confused and distributive supposition in P ⇒ A conjunction of propositions of the form "This *a* is (S) *b*" can be inferred from P.
- A term *a* has merely confused supposition in P ⇒ A proposition with a disjunctive term of the form "This *a*, or that *a* etc. . . is (S) *b*" can be inferred from P, but neither a disjunction nor a conjunction of propositions of the form "This *a* is (S) *b*" can be inferred from P.

The same applies *mutatis mutandis* to the predicate term. Notice that among the (A), (E), (I), and (O) propositional forms, merely confused supposition occurs only in predicate position (in (A) propositions). But more generally, it can also occur in subject position, such as in exceptive propositions of the form "Only *a* is b."

By applying the two groups of rules successively (first the syntactical rules and then the semantic rules), one obtains the desired result, that is, an account of the quantity of individuals involved in a given assertion, and thus of the semantics of quantifier expressions. For example, in "Every man is an animal," "man" has confused and distributive supposition and "animal" has merely confused supposition, according to the syntactical rules for "every." According to the semantic rules, this proposition asserts that "man" supposits for all of the individuals falling under it (men) and that "animal" supposits for several individuals, but not (necessarily) for all of those falling under it.

Terrence Parsons (1997) has made the compelling suggestion that the differences between the thirteenth and fourteenth century approaches can also be explained on the basis of the distinction between the study of the semantics of quantifier expressions taken individually versus the study of global quantificational effect in wider propositional contexts. Indeed, fourteenth century authors had a keen interest in the effect of nested quantifier expressions, such as the effect of a negation over an affirmative universal quantifier. Take "Not every man is an animal": according to the thirteenth century authors, "man" would have distributive and confused supposition, since it is preceded by "every." But for fourteenth century authors, the negation preceding "every" would have the effect of suppressing its distributive effect, so that "man" would no longer have distributive and confused supposition but rather determinate supposition (see Karger 1993; Dutilh Novaes 2008). In sum, "[w]hat distinguishes the earlier theory from the later one is whether the mode of supposition of a term in a proposition is something that that term retains when its proposition is embedded in further contexts" (Parsons 1997:43).

Further Developments

For reasons of space, I can only present the rough lines of the approach to quantification based on supposition. But medieval authors developed it further in several different directions, such as: the definition of valid inferences among different categorical propositions (see Karger 1993; Dutilh Novaes 2004); an analysis of multiple quantification (of subject and predicate) and of other quantifier expressions (see Ashworth 1978); discussions on what are now known as anaphoric pronouns (see Parsons 1994). Here I have discussed thirteenth and fourteenth century authors only, but fifteenth and sixteenth century authors refined the framework even further, dealing in particular with the difficulties that emerged from the earlier theories (see Ashworth 1974, 1978; Karger 1997; Dutilh Novaes 2008).

The modes of personal supposition have been a topic of heated debate in the literature, but a consensus as to their purpose and some of the technical details involved has not yet been reached. It is clear that they can be said to be a general theory of quantification, but one must bear in mind that the overall approach is fundamentally different from modern post-Fregean theories of quantification.

See also: ► John Buridan ► Peter of Spain ► Supposition Theory ► Syncategoremata ► Terms, Properties of ► William of Ockham ► William of Sherwood

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Qusṭa ibn Lūqā

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Abstract

Qusṭa ibn Lūqā was a Melkite Christian translator of Greek origin who took part in the Graeco-Arabic translation movement in the ninth century Baghdad.

Qusṭa ibn Lūqā, called *al-Tarjuman*, "the Translator," was born in Baʿalbak in contemporary Lebanon in 860. Ibn al-Nadīm in the *Fihrist* reports that he traveled to parts of the Byzantine Empire and brought back to Baghdad – where he spent most of his life – several Greek scientific manuscripts. Qusṭa ibn Lūqā was patronized by the ʿAbbāsid court; he was also in contact with al-Kindī for medical–philosophical questions, as well as with other scholars of the time: he had an epistolary exchange with Ḥunayn b. Isḥāq and the Muslim astronomer Abū ʿIsā ibn al-Munajjim, who had invited him to embrace Islam. Qusṭa ibn Lūqā's refutation of Ibn Munajjim's proof of Muḥammad's prophecy is extant. He retired in Armenia, where he died in 912.

Qusṭa ibn Lūqā was a native Greek speaker and had of course an excellent knowledge of Greek, but also of Syriac and Arabic. He translated into Arabic many Greek texts on natural science, mathematics, medicine, mechanics, and astronomy. In particular, he translated some treatises that in Late Antiquity were studied after Euclid's geometry as an introduction to Ptolemaic astronomy, and which formed together the so-called *Little Astronomy* or *Intermediate Books* (*Kutub al-mutawassiṭāt*). Among these are extant the versions of the *Spherics* (*Kitāb al-Ukar*) by Theodosius of Bithynia, the *Rising and Setting of the Fixed Stars* (*Kitāb al-Ṭulūʿ wa-l-ġurūb*) by Autolycos, and the *Lifting-Screw* (*Kitāb Rafʿ al-athqāl*) by Hero of Alexandria.

As for philosophy, Qusṭa ibn Lūqā probably took in Baghdad and translated into Arabic a copy of Pseudo-Plutarch's *Placita philosophorum*. This doxography, edited and translated by Daiber (1980) as the *Aetius Arabus*, was the main source in the Arab world of the time for the knowledge of the Presocratics and of Stoic philosophy. Qusṭa ibn Lūqā is also recorded as the translator of Aristotle's *Physics* and Alexander of Aphrodisias' and Philoponus' commentaries on it.

According to the lists of the biographers, Qusṭa ibn Lūqā wrote more than 60 original works: commentaries on Euclid; treatises on astronomy, like the extant *Book on the Use of the Celestial Globe* (*Kitāb fī l-ʿamal bi-l-kura al-nujūmiyya*) and the treatise *On the Configuration of Celestial Bodies* (*Hay'at al-aflāk*); works on medicine, as the *Medical Regime for the Pilgrims to Mecca* (*Fī tadbīr al-badan fī l-safar*) and the *Book on the Reasons Why People Differ in Their Character Traits, Their Way of Life, Their Desires, and Their Preferences* (*Kitāb fī ʿilal ikhtilāf al-nās fī akhlāqihim wa-siyarihim wa-shahawātihim wa-khtiyārātihim*); a treatise on the division of sciences, and writings on natural science. In this field, his treatise *On the Difference Between the Spirit and the Soul* (*Risāla fī l-farq bayn al-rūḥ wa-l-nafs*), in Latin translation (*De differentia spiritus et animae*), was one of the books to be read within the program on Natural Philosophy at the Faculty of Arts in Paris in 1254.

See also: ► Aristotle, Arabic ► al-Kindī, Abū Yūsuf Yaʿqūb ibn Isḥāq ► Mathematics and Philosophy in the Arab World ► Medicine in the Arab World ► Presocratics in the Arab World ► Translations from Greek into Arabic

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Quṭb al-Dīn al-Shīrāzī

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Abstract

Quṭb al-Dīn al-Shīrāzī was a thirteenth-century Persian polymath, physician, mathematician, astronomer, philosopher, and Sufi.

Quṭb al-Dīn Maḥmūd ibn Masʿūd al-Shīrāzī was born in Shirāz in October 1236 in a family of Sufi tradition. He began studying medicine under his father, who practiced and taught medicine at the Moẓaffari hospital in Shīrāz. After his father's death, his uncle and other physicians of his time trained him in medicine: Quṭb al-Dīn studied Ibn Sīnā's *Qānūn* (the *Canon*) and its commentaries including that of Fakhr al-Dīn al-Rāzī. Still young, he served at the Moẓaffari hospital as an ophthalmologist.

Some time after 1260, Quṭb al-Dīn left Shiraz for Maragha, where Naṣīr al-Dīn al-Ṭūsī had established the famous observatory, a point of attraction for many scholars from all over the country. In Maragha, Quṭb al-Dīn studied astronomy as well as Ibn Sīnā's philosophy and medicine. He read under Naṣīr al-Dīn al-Ṭusī Ibn Sīnā's *Ishārāt wa-l-tanbīhāt* (*Pointers and Reminders*) and the *Kulliyāt* of the *Qānūn*. He moved then to Khorasan, and decided to study and work in Juwayn with Najm al-Dīn ʿAlī ibn ʿUmar al-Qazwīnī al-Kātibī, a cofounder of the observatory in Maragha.

Quṭb al-Dīn traveled a lot, and some time after 1268 he went to Qazwīn, Iṣfahān, and Baghdad; later on, he traveled to Konya in Anatolia, where he probably met the famous Persian poet Jalāl al-Dīn Muḥammad Balkhī (al-Rūmī). Then, Quṭb al-Dīn was named by the governor of Konya judge of Sivas and Malatya.

In 1282, the Mongol Il-khān Aḥmad Takudār sent Quṭb al-Dīn to the Mamluk ruler of Egypt. Quṭb al-Dīn spent the rest of his life teaching Ibn Sīnā's philosophy and medicine. He died in Tabriz in 1311 and was buried in the Čarandāb cemetery.

The encyclopedic knowledge of Quṭb al-Dīn is well documented by his Persian and Arabic writings: he wrote on philosophy following the illuminationist tradition, as well as on medicine, mathematics, geometry, astronomy, geography, Sufism, theology, law, linguistics, and rhetoric. Among his works are the *Durrat al-tāj* (*The Pearly Crown*), an encyclopedic work on philosophy, natural science, theology, logic, astronomy, mathematics, and music written in Persian around 1306 for the ruler of the Iranian land of Gilan; the *Sharḥ Ḥikmat al-Ishrāqī Shaykh Shihāb al-Dīn al-Suhrawardī* (*Commentary on al-Suhrawardī's Philosophy of Illumination*) written in Arabic; the *Nuzhāt al-ḥukamā' wa-rawḍat al-aṭibbā'*, a comprehensive commentary in five volumes on Ibn Sīnā's *Kulliyāt* written in Arabic; the *Eḳtiārāt-e moẓaffari*, a treatise on astronomy in Persian; the *Nihāyat al-idrāk fī dirāyat al-aflāk* (*The Limit of Accomplishment Concerning Knowledge of the Heavens*), where Quṭb al-Dīn describes the planetary motions, improving Ptolemy's model and advancing the possibility for heliocentrism.

See also: ► Fakhr al-Dīn al-Rāzī ► Ibn Sīnā, Abū ʿAlī (Avicenna) ► Medicine and Philosophy ► Naṣīr al-Dīn al-Tūsī ► al-Suhrawardī, Shihāb al-Dīn Yaḥyā al-Maqtūl

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